A New Topology Optimization Methodology Based on Constraint Maximum-Weight Connected Graph Theorem and Support Vector Machine

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To eliminate the checkerboard pattern, a new topology optimization methodology based on the constrained maximum-weight connected graph theorem and algorithm is proposed. A Kriging model based on the support vector machine for predicting the sensitivity is introduced to reduce the overwhelmingly heavy computational burden as required in a topology optimizer. From the numerical results as reported, it is observed that the proposed methodology is able to avoid the checkerboard pattern, and can improve significantly the final solutions with a reduced computation time.

***Index term*—Checkerboard pattern, constrained maximum-weight connected graph algorithm, support vector machine, topology optimization**

I. A New Topology Optimization Methodology

Topology optimization is the conceptual design of a product and also the highest level in the design phase. It can create a novel topology, which is beyond ones imagination, for a product in its early conceptual design phase. Nowadays, topology optimization has become the new paradigm to provide a quantitative design method for modern devices [1].

A topology optimization is generally defined to find the best material distribution of the structure in the design space, and it is only after the introduction of computationally expensive finite element methods into topology optimization procedures that the topology optimization techniques can be used to solve various types of structure topologies. Moreover, the gradient information is generally employed to update the material of an element. All these factors are demanding heavy computation resources to make topology optimization an overwhelmingly complicated numerical study. Moreover, checkerboard patterns and gray areas are often involved in the final solutions of a topology optimizer. To address these two issues, a new topology optimization methodology based on the constraint maximum-weight connected graph (CMWG) theorem is firstly proposed and validated.

A. Iterative Procedures of the Proposed Methodology

To facilitate the understanding and description of the proposed methodology, its iterative procedures are given as:

Step 1: Compute the sensitivity of every element in the search domain;

Step 2: Apply the CMWG theorem and algorithm to update the state of every element to generate new checkerboard pattern free candidate topologies;

Step 3: Annealing? If No, go to Step 1; Otherwise, continue to the next Step;

Step4: Anneal. Stop Iterations? If Yes, terminate the methodology; Otherwise, go to Step 1.

B. Element Attribute Updating using CMWG Theorem

A checkerboard pattern refers to the phenomena of alternating presence of solid and void elements ordered in a checkerboard like fashion [1]. This pattern can be commonly produced in various finite element based structural optimization processes. In order to eliminate the checkerboard patterns, the CMWG theorem and algorithm are firstly extended to update the ON/OFF state of an element.

Given a positive integer R and an undirected graph G=(V,E), in which each vertex is assigned a weight, the CMWG theorem and algorithm are to find a connected subgraph with R vertices that maximizes or minimizes the sum of the weights under the constraint of having some predefined vertexs included in the solution [2].

To implement CMWG algorithm in topology optimizations, the integer R is the number of elements that should be added and updated to the currently optimized elements, and the weight in an element is the sensitivity of the element in issue. To intuitively explain the mechnism of the proposed CMWG algorithm to update the attribute of an element, a step by step description of R=2 using Fig.1 and Fig.2 is given as:

Step1: Identify the sub-region to update element attributes.

To identify the sub-region, for a 2D problem, for every element, a rectangle, centered in the element in question with a side length of (2R+1), is firstly constructed. In Fig. 1(a), the green region is the identified sub-region from one fixed (yellow) element. Morover, Fig.2 (a) shows the corresponding sub-region which is a union of all rectangles identified from each fixed element when there are more than one fixed elements.

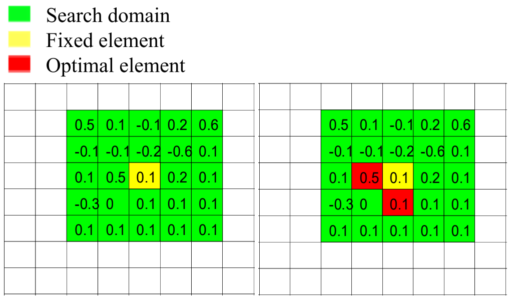
Step2: Find a connected subgraph of R elements in the search (just identified) domain under the constraint of having a predetermined vertex included.

There are 276 (C_{24}^{2}) possible two-element combinations in the search domin as illustrated in Fig.1(a); while the two red elements with the maximum sum of weights are selected as the optimal elements to update element states under the constraint of continuously connecting with the yellow element (air element) as shown in Fig.1 (b).

Since the CWMG problem is NP-hard, for a large scale graph, it is not feasible to solve the integer linear programming directly. For this reason, the Balas additive method [3], an enumeration scheme for solving 0-1 integer programming problem, is employed in this paper.

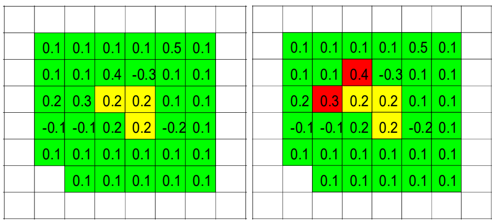
Step3: Update the fixed elements and return to step1

After the two red elements are selected as the optimal elements in Fig.1(b), the fixed elements are updated as the yellow colored region of Fig.2 (a) containing three elements. The procedure continues to next cycle starting from the topolgy as shown in Fig.2 (b).



(a) (b)

Fig. 1. Element states in the 1st step in the proposed methodology



(a) (b)

Fig. 2. Element states in the 2nd step in the proposed methodology

C. Sensitivity Computation

To determine the material attribute of an element, the sensitivity is used in the proposed methodology. Nevertheless, the computational cost of the proposed methodology is still overwhelmingly expensive because of nearly infinite decision parameters. To address this issue, a fast and accurate approach is proposed based on a support vector machine (SVM). More specially, to reduce the computational burden for sensitivity analysis, after every M cycles, the sensitivity of an element is predicted from the information gathered both in its previous n iterations and in its neighbor elements, which are defined in Fig.3. It should be pointed out that to consider the different contributions of an element in issue and its neighbor ones, the sensitive information for the two type elements is a weighted accumulation of the information.

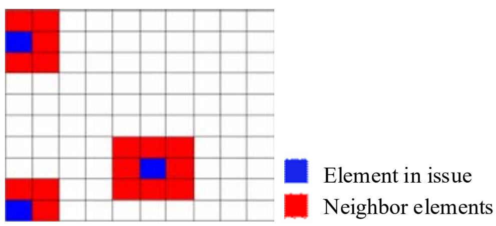


Fig.3. The definition of three types of element neighbors

II. Numerical Application

To testify the proposed methodology, a magnetic actuator including a yoke, a coil and an armature is topologically optimized to maximize the magnetic force in a specific direction.

To predict the sensitivity using the proposed SVM based Kriging model, the history data in the previous 10 iterations and neighbor elements, a total of 10×6 training data, are used to compute the sensitivity of every element after every 2 cycles. The details of the 10×6 matrix training data are explained as: each row represents the index of the element. The 6 columns represent, respectively, the sensitivity of the element; the average, the maximum, the mimimum sensitivities of the neighbor elements; the force and the material volume percentage of the present topology. Fig. 4 compares the real sensitivity and the predicted result of the 10^{th} iterartive cycle. Table I tabulates the final solutions of the ON/OFF method and the proposed methdology with (CMWG_with) and without (CMWG_no) the proposed sensitivity computation methodology. It should be noted that in order to avoid the checkerboad patterns, a very complex annealing mechnism is used in the ON/OFF method. Moreover, the checkerboad patterns as commonly accompnay the final solution of a topolgy optimizer is effectively avoided.

To summary, the constraint maximum-weight connected graph theorem and algorithm are firstly extended to devlop a novel topology optimization methdology to eliminate the checkerboad patterns. A suport vector based Kriging model is introduced for the fast computation of the sensitivity.

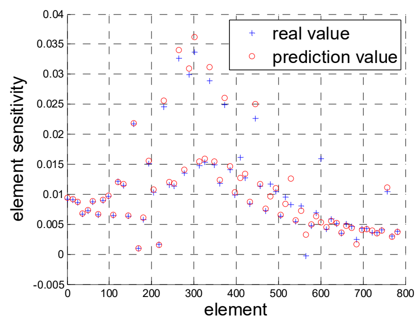


Fig. 4. The real and predicted sensitivities in the 10^{th} cycle.

TABLE I
THE COMPARISON OF DIFFERENT TOPOLOGY OPTIMIZATION METHODS

	ON/OFF	CMWG_no	CMWG_with
Force (N/m)	55.82	54.16	54.16
Material volume(%)	75	74	74
No. iterations	/	49	33

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